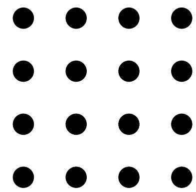
PROBLEMS on a SQUARED PAPER

1. Draw six segments as one continuous line so that they pass through all of the 16 nodes forming a square 4×4.



2. Is there a right triangle with integer sides such that all of its vertices are nodes, but none of its sides lies on a line of a squared paper? What if one side of this triangle should lie on a line of a suared paper?

3. Five nodes of a squared paper are marked. Proove that there are two of them such that their bisecting point is also a nod of the lattice.

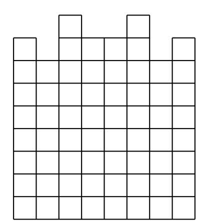
4. Gardeners live in nodes of a squared paper. Flowers grow everywhere. Every flower is looked after by three gardeners closest to it. Draw the area for a certain gardener to look after.

5. On a squared paper draw a triangle, two medians of which are perpendicular to each other.

6. Can there be a hole in a notebook sheet such that a man can pass through it?

7. In every little square of a square 8×8 one of the diagonals is drawn. Consider the union of these 64 diagonals. It consists of several connected parts. Can the number of these parts be **(a)** greater than 15? **(б)** greater than 20?

8. Cut this figure into two pieces and put the pieces together in such a way to form a square $8{\times}8$.



9. What is the least number of chess kings that can be put on a chess board so that no two of them attack each other, but an extra king at any other square, if put on the board, will attack one of these kings? (A king is a piece that can move in any direction by exactly one square.)

10. Given a squared rectangle 100×101, find into how many pieces its diagonal is split by the grid.

11. From a squared paper, a square $M \times M$ is cut. Then a smaller square $N \times N$ is cut from the $M \times M$-square. There are 124 little squares left. Find all possible values of M and N.

12. Given a rectangle pool 2013×2014, the ball is stroke frome a hole at the angle of $45°$ according to the edges. Find the number of reflections before the ball falls into a hole. What hole will that be? (Incoming angle is equal to outgoing.)

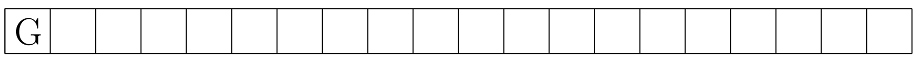
13. A grid splits a square into 25 little squares. In some of the little squares one of the diagonals is drawn so that there are no diagonals with common points. What is the maximal amount of the diagonals drawn?

14. Given 64 points forming a square 8×8 points, draw the minimal number of lines, not parallel to the sides of the square, such that they cover all 64 points.

15. Given 64 points forming a square 8×8 points, draw the minimal number of lines, not passing through these points, separating the points from each other.

16. Given a grid 11×11, call the segment connecting two neighbor nodes *an edge*. Erase the minimal number of edges so that every node be incident to not more than 3 edges.

17. A grasshopper is jumping along a squared stripe of length 20. It start from the left end and jumps for 1 or 2 squares **to the right** each time. Find the number of different ways for the grasshopper to reach the last square on the right.



18. **(a)** How namy ways are there to cut a rectangle 8×2 into rectangles 1×2? **(б)** What about a rectangle $n \times 2$?

19. Lame king is a chess piece that can only move either one position down or one position to the right. How many ways are there for a lame king to get from $a8$ to **(а)** $c6$? **(б)** $d7$? **(в)** $d5$? **(г)** $h1$? (Standard chess notation.)

20. Given a squared paper with unit length squares, find the area of the figure:

21. Four grasshoppers sit in the vertices of a square. Every minute one of them jumps to a point symmetrical to his current place with respect to one of the other grasshoppers. At some moment it occures that the grasshoppers form a square again. Prove that its side is equal to the side of the original square.

22. A convex polygone is drawn on the squared paper so that its vertices lie in the nodes of the grid and none of its sides lies on the grid lines. Prove that the sum of lengths of the horizontal grid lines inside the polygon is equal to the sum of lengths of the vertical grid lines inside the polygon.

23. **Pick's fomula.** Vertices of a polygon are in nodes of a grid. Inside it there are exactly n nodes of the grid, on its sides there are exactly m nodes. Prove that the area of the polygon is equal to $n + m/2 - 1$. (The polygon is not necessarily convex.)

24. A triangle ABC lies on the grid in such a way that there are no nodes on its sides and there is only one node O inside it. Prove that O is the weight center (intersection of medians) of the triangle ABC.