1. Can a 10×10 square board be dissected along grid lines into 11 rectangles, no two of which are congruent?

2. There are 2011 representatives from three tribes along a circle: knights, liars and conformists. Knights always tell the truth, liars always lie, while a conformist lies only if it stands next to a liar (in any case he can tell the truth). Each of them says: “My neighbors come from different tribes.” What is the smallest possible number of liars among them?

3. Eight numbers are written on a blackboard. Two persons alternately play, and in each step two different numbers are replaced with two equal numbers with the same sum. The first player wins if, at some moment, the eight numbers can be partitioned into two groups of four with equal sums. Can the second player prevent him from winning?

4. There are several (more than 3) towns in a country, and some of them are connected by roads. Suppose that, if any town is excluded, one can still travel between any two cities without passing through the excluded town, but this condition is no longer fulfilled if any road is closed. Prove that no three of these towns are pairwise connected by roads.

5. In some cells of an infinite square row there is a black or white piece, while in the others there is none. There are finitely many pieces in total, at most one in each cell. We repeatedly perform one of the following two operations:

(i) If consecutive cells A, B, C (in this order) are such that there are pieces of different colors in A and C, then we can remove a piece from B if there is any, or put a piece of an arbitrary color in it otherwise.

(ii) If there is a piece in each of the consecutive cells A, B, C, D (in this order), and the pieces in A and D have the same color, then we can exchange the pieces in B and C.

Prove that, by these operations, we cannot change the color of a piece lying between two pieces of the same color, so that the content in every other cell is the same as at the beginning.

1. A rectangular board with an area greater than 1 is given. Show that it is possible to mark some (but not all) of its cells so that each unmarked cell has exactly one marked neighboring cell (sharing a side).

2. In a country, there are n provincial towns and the capital city. The capital is connected with all other towns by direct airlines. Moreover, there are direct airlines between some provincial towns, and between any two provincial towns there is exactly one route not going through the capital (and not visiting the same town twice, but with possible transfers in other provincial towns). The government wants to make each of these airlines one-way so that, after departing from any town, it is not possible to return to that town. In how many ways can the government achieve this? (For example, for $n = 2$ there are 6 ways.)

3. Every point on a circle is painted in one of 100 given colors. Show that there always exists a trapezoid that is inscribed in the circle and whose all vertices are of the same color.

4. There are 18 weights with masses from 1 to 18 grams. They are marked with numbers from 1 to 18, denoting mass, but two of the marks have been exchanged. Can we always determine the exchanged pair with 4 measurings on a calibrated scale (showing the total mass of weights being measured)?

5. A group of several people is said to be *connected* if it cannot be partitioned into two nonempty subgroups so that no two persons in different subgroups are unacquainted. In a connected group, every person knows exactly four others, and the four acquaintances of each person form a connected subgroup. Prove that the people in this group can be ordered around a circle so that every two neighbors are acquainted.