1.1 Solve the system of equations $x + yz = y + zx = z + xy = 6$.

1.2 Let P be a point on side BC of a square $ABCD$. Construct square $APRS$ on segment AP, of the same orientation as $ABCD$. Prove that the angle RCD equals $45°$.

1.3 Show that one can dissect a 16×16 square onto rectangles 1×4 in such a way that no lattice point belongs to four distinct rectangles.

2.1 What is the largest n for which there exist n consecutive natural numbers whose product ends with the digits 3000?

2.2 In an isosceles triangle ABC ($AB = AC$) the angle BAC equals $40°$. Points S and T on sides AB and AC respectively are such that $\angle BAT = \angle BCS = 10°$. Segments AT and CS intersect at point P. Prove that $BT = 2PT$.

2.3 Two players alternately draw red and blue lines in the plane, which do not pass through intersection points of other lines and are not parallel to other lines. In each move, each player can arbitrarily choose the color of his line. The game ends when each player draws 20 lines. The second player wants to produce as many intersection points of lines of distinct colors as possible; the first player attempts to spoil his efforts. What is the largest number of such intersection points that the second player can always achieve?

3.1 Let $P(x)$ be a quadratic trinomial with integer coefficients (and a nonzero leading coefficient) such that $P(1) = 2011$ and $P(2011) = 1$. Can there exist an integer m for which $P(m) = m$?

3.2 Let ABC be an isosceles triangle with base BC and let BD be its angle bisector. Given that $BD + DA = BC$, find the angles of triangle ABC.

3.3 There are 100 apparently equal coints, but it is known that exactly 4 of them are fake. The fake coins weigh the same and are lighter than other coins. How to find at least 13 genuine coins by 2 measurings on a scale without weights?

4.1 The natural numbers from 1 to 100 are written in cells of a 10×10 square board in some order. Consider the products of numbers in each row and in each column. Can the largest of the row products be divisible by each of the column products?

4.2 Let h_a and h_b be the altitudes on the non-parallel sides a and b of a parallelogram $ABCD$, respectively. Suppose that $a + h_a = b + h_b$. At most how many different lengths can there be among AB, AC, AD, BC, BD, CD?

4.3 On an island, only liars (who always lie) and knights (who always tell the truth) live. To the question "Is the number of knights who are your friends even?" everybody answered "No". To the question "Is the number of liars who are not your friends even?" everybody answered "Yes". Is the number of people on the island even or odd?

1.1 Solve in real numbers the equation $x\sqrt{y-1}+y\sqrt{x-1}=xy$.

1.2 In a regular pyramid $ABCDS$ with the top vertex S, the length of AS equals 1 and angle ASB equals $30°$. Determine the length of the shortest path from A to A that meets all lateral edges except AS.

1.3 A square board $N \times N$ is cut along the grid lines into polygons whoes areas do not exceed k ($k > 3$). Suppose there is a polygon that is neighboring to all others. Prove that $N < k\sqrt{3}$.

2.1 A natural number $k > 1$ is given. If a, b, c are natural numbers such that a divides b^k, b divides c^k, and c divides a^k, what is the smallest $n = n(k)$ for which it is always true that $abc \mid (a+b+c)^n$?

2.2 In a quadrilateral $ABCD$ with $\angle A = 60°$, $\angle B = 90°$, $\angle C = 120°$, the point M of intersection of the diagonals satisfies $BM = 1$ and $MD = 2$. Find the area of quadrilateral $ABCD$.

2.3 A *lame scale* is a scale without weights which breaks after only two instances of imbalance (when its two sides have different weights) and cannot be used anymore. One of N coins is fake, lighter than the others. What is the greatest N for which one can find the fake coin with k measurings on a lame scale?

3.1 Find all real numbers α for which the system of equations

$$\frac{a^3}{b+c+\alpha}=\frac{b^3}{c+a+\alpha}=\frac{c^3}{a+b+\alpha}$$

has a solution in distinct real numbers a, b, c in $[-1,1]$.

3.2 Let A, B, C be points on a line with B between A and C, $AB = 3$ and $BC = 5$. If BMN is an equilateral triangle, find the smallest possible value of $AM + CN$.

3.3 Let $n \geqslant k$ be natural numbers. There are n sections in a school, and for any k sections every student belongs to at least one of them, but this does not hold for any $k-1$ sections. What is the smallest possible number of students in this school?

4.1 Solve in (positive) prime numbers the equation $p^2 + pq + q^2 = r^2$.

4.2 In a cyclic hexagon $ABCDEF$ it holds that $AB = BC$, $CD = DE$ and $EF = FA$. Show that $S_{ABCDEF} = 2S_{BDF}$.

4.3 Let n be a natural number. For which natural numbers $k \leqslant n$ can one write numbers in the cells of an $n \times n$ board such that the sum of all numbers in the board is positive, but the sum of numbers in any $k \times k$ square is negative?