1. (4) There are 11 coins in a row, and the weights of any two neighboring coins differ by 1 gram. It is known that, among the coins that are not outermost, one has both neighbors lighter than itself, while all others have one neighbor lighter and one heavier than itself. How can one find the heaviest coin in 2 measuring on a scale without weights?

2. (5) In a triangle ABC, the angle bisector at A meets the opposite side at L. Suppose that $AB = 2007$ and $BL = AC$. Determine the side lengths of triangle ABC, given that they are integers.

3. (5) The students in a school attend some sections. Show that we can always promote some students into pioneers in such a way that in each section there is at least one pioneer, and for each pioneer there is a section in which he is the only pioneer.

4. (6) We call a natural number *interesting* if it can be written in the form $a^2 + 2011b^2$ for some natural numbers a, b. Prove that if p is a prime and p^2 is interesting, then at least one of the numbers p and $2p$ is interesting as well.

5. (8) Numbers a, b, c lie in the segment $[2, 4]$. Prove the inequality

$$\frac{2}{a+b^2+c^3}+\frac{2}{b+c^2+a^3}+\frac{2}{c+a^2+b^3}\leqslant\frac{3}{a+b+c}.$$

6. (10) The natural numbers from 1 to 1000 are written on the blackboard. Petya and Vasya play alternately, with Petya playing first. In each move, some two numbers are replaced with their sum. When only two numbers remain on the blackboard, if one is divisible by the other, then Vasya wins, otherwise Petya wins. Who has a winning strategy?

7. (10) Points P and Q on the side AB of a convex quadrilateral $ABCD$ are such that $AP = QB$. Point X ($X \neq D$) is an intersection point of the circumcircles of triangles APD and DQB, and Y ($Y \neq C$) is an intersection point of the triangle ACP and QCB. Prove that the points C, D, X and Y lie on a circle.

8. (12) A regular hexagon is divided into 54 congruent equilateral triangles (see the picture). The 37 vertices of these small triangles are arbitrarily denoted with natural numbers from 1 to 37. We call a small triangle *good* if, going along its sides from the smallest number to the largest one, we move clockwise. Prove that there must be at least 19 good triangles.

1. (3) Solve the system of equation in real numbers:

$$a^2 + b^2 = 2c \quad 1 + a^2 = 2ac \quad c^2 = ab.$$

2. (4) The cells of a 50×50 square board are painted in 50 colors, with exactly 50 cells of each color. Prove that there is a line (a row or a column) containing not less than 8 distinct colors.

3. (6) For a given natural number n, how many positive integer solutions does the equation $3x^2 + 5y^2 = 2^n$ have?

4. (6) In a triangle ABC, the segment AL is the angle bisector, and I_1 and I_2 are the incenters of triangles ABL and ACL respectively. Line I_1I_2 intersects the sides AB and AC at points C_1 and B_1 respectively. Prove that the lines BB_1, CC_1 and AL meet at one point.

5. (7) A circle of area 1 is given. For a set of points A within the circle and a diameter d of the circle, we denote by A_d the set symmetric to A with respect to d. Does there exist a set A of area $1/2$ within the circle, such that the intersection $A \cap A_d$ has the area $1/4$ for each diameter d?

6. (7) Let a, b, c be numbers from the segment $[1, 2]$. Prove the inequality

$$\frac{1}{1+a+b^2} + \frac{1}{1+b+c^2} + \frac{1}{1+c+a^2} \leqslant \frac{3}{a+b+c}.$$

7. (8) Find all functions $f : [0, +\infty) \to \mathbb{R}$ such that $f(x + f(x) + 2y) = f(2x) + 2f(y)$ for all nonnegative x, y, and the equation $f(x) = 0$ has finitely many solutions (or none).

8. (9) Consider a triangle ABC and concentric circles ω_b, ω_c centered at A. An arbitrary ray from A intersects ω_b and ω_c at points B' and C' respectively. The perpendicular bisectors of segments BB' and CC' meet at point X. Prove that, as the ray from A varies, all such points X lie on a line.

9. (10) Let B_k be the number of ways to partition a k-element set into nonempty subsets (for example, $B_3 = 5$, because the set $\{1, 2, 3\}$ has 5 possible partitions: (123), (1,2,3), (12,3), (13,2), (1,23)). Prove that if p is a prime and k a natural number, then $B_{k+p^p-1} - B_k$ is divisible by p.