1. In a triangle ABC, AA_1 is the median and M the centroid. Point K on side BC is such that $MK \parallel AC$. If $AM = CK$, determine the angle ACB.

2. Given 2011 points in the plane, we call a pair of points A, B *isolated* (the order is irrelevant) if the circle with a diameter AB contains no other given points. What is the smallest possible number of isolated pairs?

3. In a triangle ABC, points M and L on side BC are the midpoint and the foot of the bisector of angle A, respectively. Points P and Q are the orthogonal projections of point L on sides AB and AC, respectively. Point X on the median AM is such that $XL \perp BC$. Prove that the points P, X and Q lie on a line.

4. Let $ABCD$ be a convex quadrilateral for which $AB + CD = \sqrt{2} \cdot AC$ and $BC + DA = \sqrt{2} \cdot BD$. Prove that $ABCD$ is a parallelogram.

1. In a rectangle $ABCD$, P is the midpoint of side AB and Q is the foot of the perpendicular from C on PD. Show that $BQ = BC$.

2. A tetrahedron $ABCD$ is given. A sphere through points A, B and C intersects the lateral edges DA, DB, DC again at points A_1, B_1, C_1, respectively. Points A_2, B_2, C_2 are the respective reflections of points A_1, B_1, C_1 across the midpoints of the corresponding edges. Prove that the points A, B and C are equidistant from the center of the circumscribed sphere of tetrahedron $DA_2B_2C_2$.

3. Points A, B, C, D lie on a circle in that order, where AB and CD are not parallel. The length of arc AB containing C and D is twice the length of arc CD not containing A and B. Let E be a point satisfying $AC = AE$ and $BD = BE$. Suppose that the perpendicular from E on the line AB passes through the midpoint of the arc CD not containing A and B. Determine $\angle ACB$.

4. Given 2011 points in the plane, we call a pair of points A, B *isolated* (the order is irrelevant) if the circle with a diameter AB contains no other given points (in its interior or on the boundary). What is the greatest possible number of isolated pairs?