

Marking Scheme

Problem 1. Unfinished calculation in coordinates etc. — 0 points.

Proof that if $AP \cdot PC = BQ \cdot QD$ then $AC = BD$ (and equivalent statements) — 4 points.

Problem 2a). Correct example — 3 points.

“Almost correct” example which differs from the correct one by cubic monomials only — 2 points.

Problem 2b). Full solution — 4 points.

Presentation of some obvious observations (such as $p(x) \pm q(x)$, $p(x)q(x)$ are good if $p(x)$ and $q(x)$ are good, etc.) — 0 points.

Reducing polynomial $3x + 3x^7 + 3x^{2008}$ to another one (e.g. to $3x^{2008}$ or to $3x$) — 0 points.

Observation that the sum $\sum_{3 \nmid i} b_i$ should be even, though without proof of this observation — 1 point.

Problem 3. It is proved that $|X| \leq 7!$ — 2 points.

Examples of sparse sets with cardinality much less than $7!$ — 0 points.

The idea to use the sum $a_1 + \dots + a_6$ (modulo some number) as the 7th term — 1 point.

Marking Scheme for Problems 5 and 6

Problem 5. It is proved that the circle B_1LB_2 touches KB_1 and KB_2 — 1 point.

It is proved that $KL \perp O_1O_2$ — 3 points.

It is shown that the statement follows from $O_1O_2 \perp KL$ — 2 points.

It is shown that the statement follows from the fact that the point P lies on circle $KA_1O_1B_1$ (or from similar statements) — 1 point.

Problem 6. The first step of the official solution (by the rearrangement inequality or by another way) — 1 point.

Proof of the inequality in the case $a \geq b \geq c$ — 1 point.

Referring to some classical theorems (e.g. Muirhead inequality) but not solving the problem — 0 points.