

Mathematical Olympiads and their goals

*Teachers College, Columbia University
New York, November 18-20, 2016*

Vladimir Sharich / Шарич Владимир Златкович

sharich@mathschool.ru

«Foxford.ru»



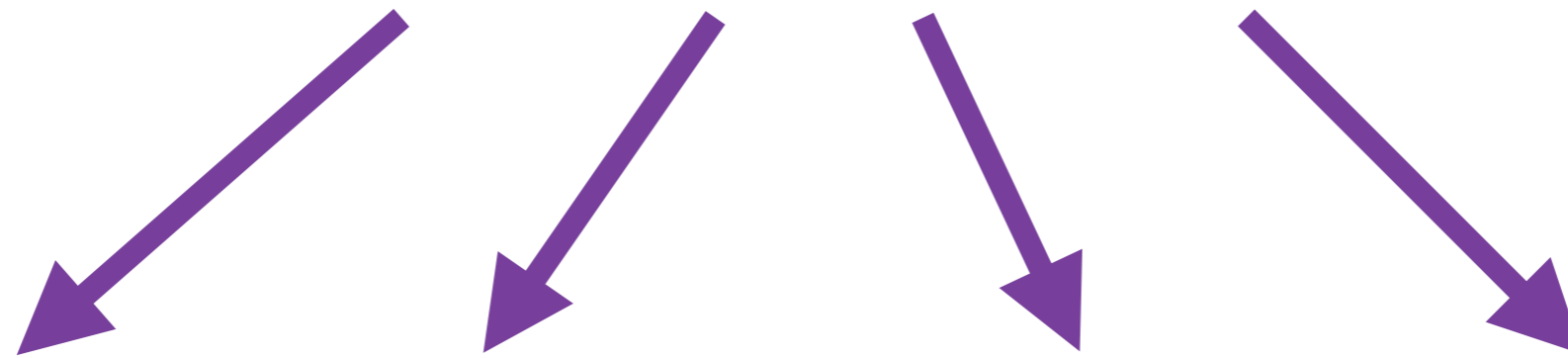
«MathSchool.ru»



«Phystech-lyceum»



Mathematical Olympiads

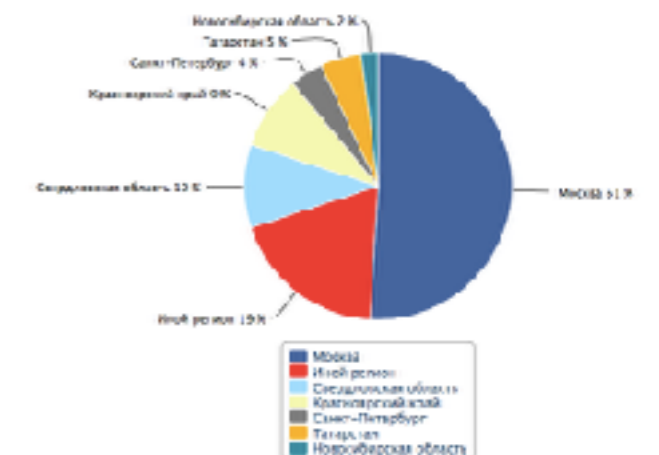
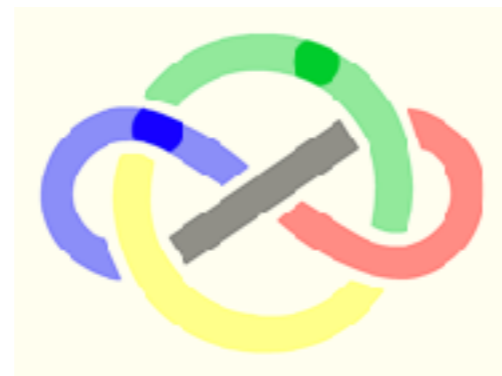
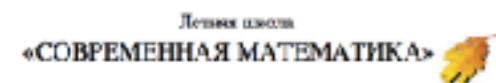
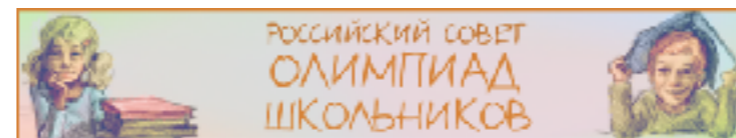
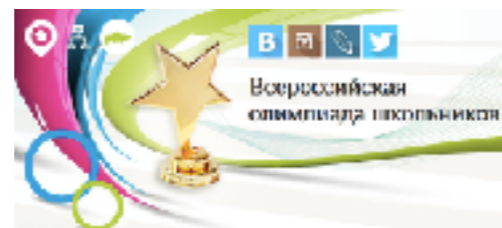


Science

Sport

University admission

Status



Mathematical Olympiads

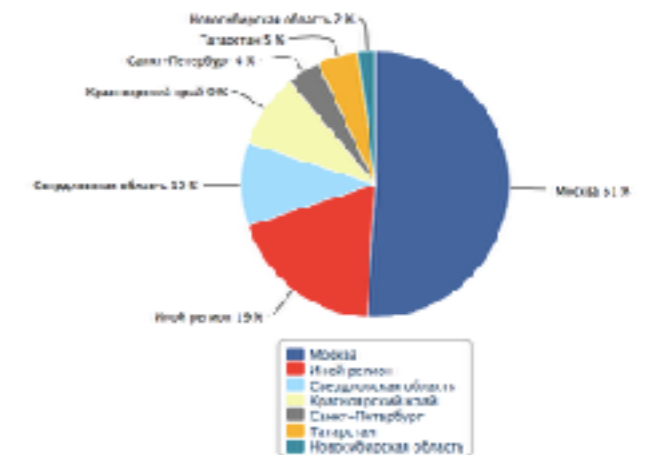
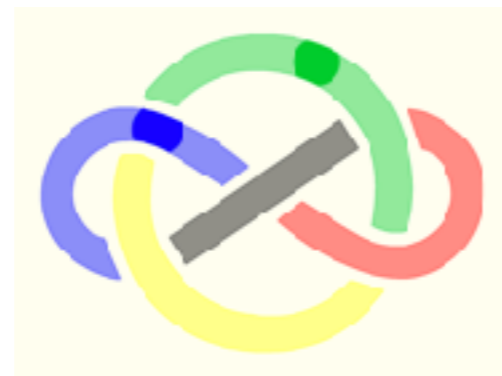
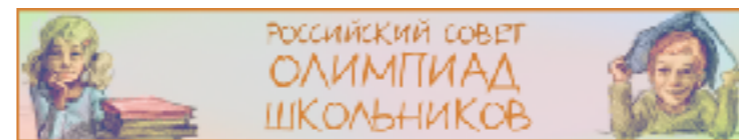


Science

Sport

University admission

Status



Plan

1. A brief history
2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



A brief history

1. **A brief history**
2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



A brief history

1. XV-XVIII century: One-vs-one

1. A brief history

2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis

4. A special experience

5. Ways to overcome the crisis



A brief history

1. XV-XVIII century: One-vs-one
2. XIX century: Mass contests

1. A brief history

2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



A brief history

1. XV-XVIII century: One-vs-one
2. XIX century: Mass contests
3. XX century:

1. A brief history

2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



A brief history

1. XV-XVIII century: One-vs-one
2. XIX century: Mass contests
3. XX century:
 - A. Town contests

1. A brief history

2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



A brief history

1. XV-XVIII century: One-vs-one
2. XIX century: Mass contests
3. XX century:
 - A. Town contests
 - B. National contests

1. A brief history

2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



A brief history

1. XV-XVIII century: One-vs-one
2. XIX century: Mass contests
3. XX century:
 - A. Town contests
 - B. National contests
 - C. World contests

1. A brief history

2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



A brief history

1. XV-XVIII century: One-vs-one
2. XIX century: Mass contests
3. XX century:
 - A. Town contests
 - B. National contests
 - C. World contests
4. XXI century: University admission contests (?!)

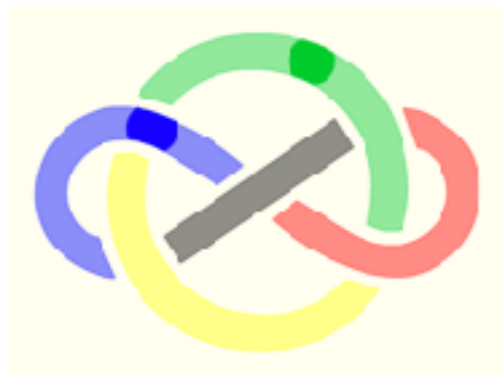
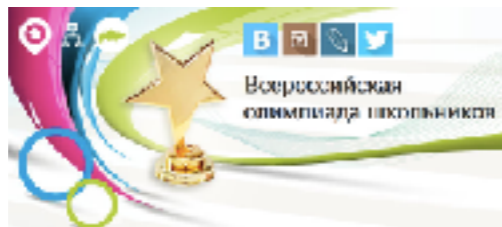
1. A brief history

2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

1. A brief history
2. Current crisis
 - A. Substantive crisis**
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

- Very few, almost none new ideas for the contest problems

1. A brief history

2. Current crisis

A. Substantive crisis

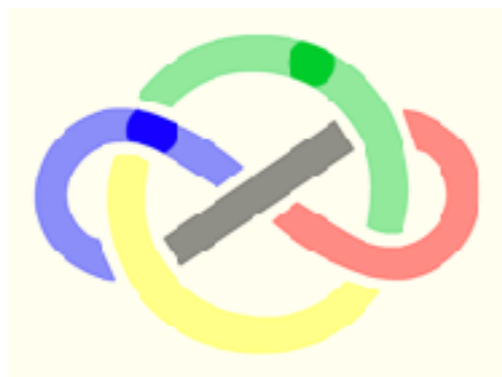
B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis

4. A special experience

5. Ways to overcome the crisis



Substantive crisis

- Very few, almost none new ideas for the contest problems
- Olympiads turned into a sport of combining standard ideas

1. A brief history

2. Current crisis

A. Substantive crisis

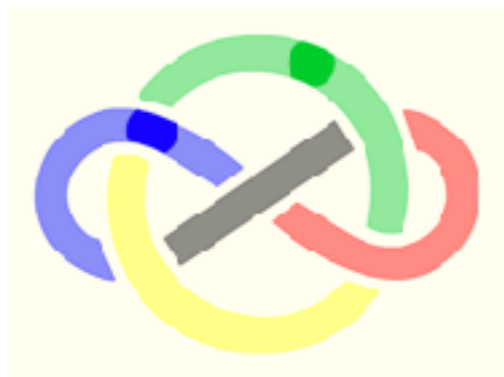
B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis

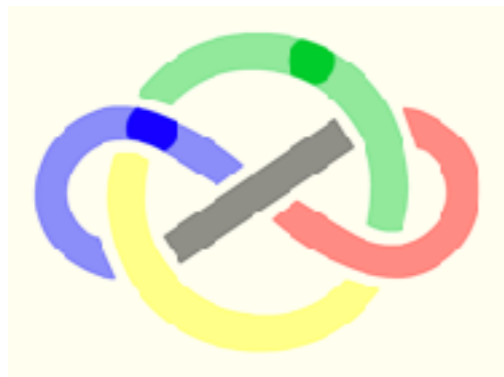
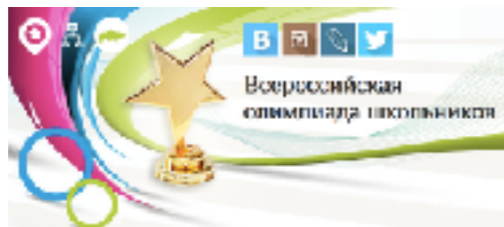
4. A special experience

5. Ways to overcome the crisis



Substantive crisis

- Very few, almost none new ideas for the contest problems
- Olympiads turned into a sport of combining standard ideas
- Excluding real science from mathematical circles



1. A brief history
2. Current crisis

A. Substantive crisis

B. Format crisis

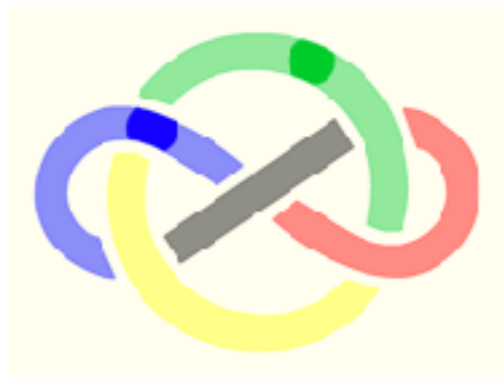
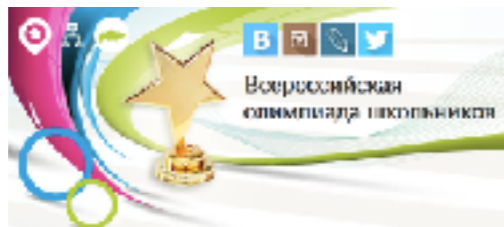
C. Ethical crisis

3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

- Very few, almost none new ideas for the contest problems
- Olympiads turned into a sport of combining standard ideas
- Excluding real science from mathematical circles



1. A brief history
2. Current crisis

A. Substantive crisis

B. Format crisis

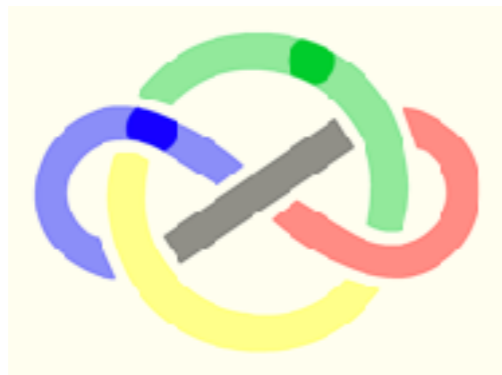
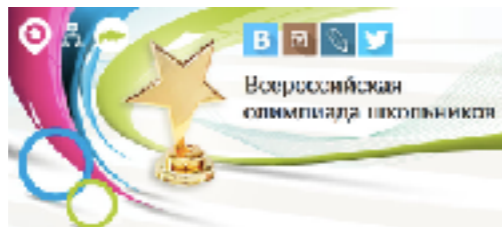
C. Ethical crisis

3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

1. A brief history
2. Current crisis
 - A. Substantive crisis**
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

- **All-Union, 1962, grade 9 of 10:** Given a square table $N \times N$ with odd N , its each cell containing ± 1 , for every horizontal and vertical row we calculate the product of the numbers in this row. Prove that the sum of obtained $2N$ numbers is not equal to 0.

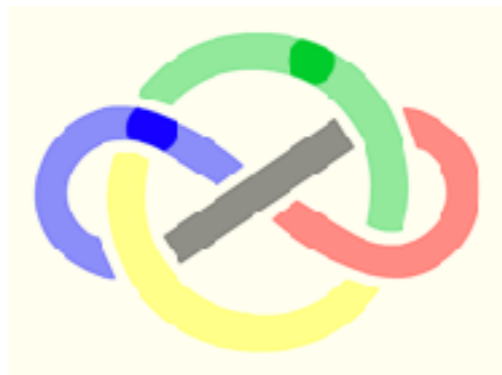
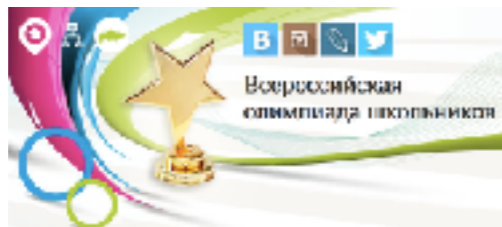
1. A brief history
2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

- **All-Union, 1962, grade 9 of 10:** Given a square table $N \times N$ with odd N , its each cell containing ± 1 , for every horizontal and vertical row we calculate the product of the numbers in this row. Prove that the sum of obtained $2N$ numbers is not equal to 0.
- **All-Russian, 2015, grade 9 of 11:** Given $N > 8$ different non-negative numbers, each less than 1, with the property that for every 8 numbers there exists 9th such that the sum of these nine numbers is integer, determine how many numbers could there be at least?

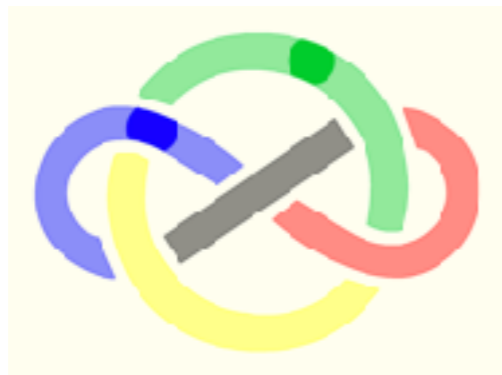
1. A brief history
2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

- **All-Union, 1962, grade 9 of 10:** Given a square table $N \times N$ with odd N , its each cell containing ± 1 , for every horizontal and vertical row we calculate the product of the numbers in this row. Prove that the sum of obtained $2N$ numbers is not equal to 0.
- **All-Russian, 2015, grade 9 of 11:** Given $N > 8$ different non-negative numbers, each less than 1, with the property that for every 8 numbers there exists 9th such that the sum of these nine numbers is integer, determine how many numbers could there be at least?

1. A brief history

2. Current crisis

A. Substantive crisis

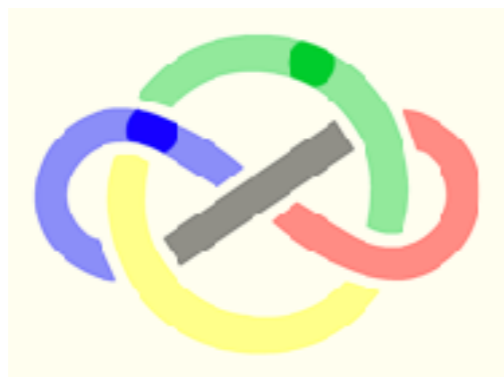
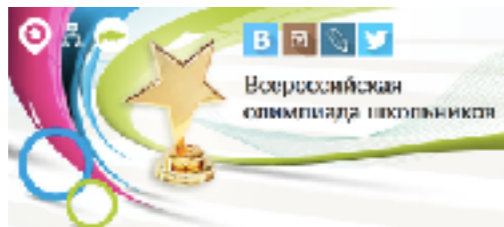
B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis

4. A special experience

5. Ways to overcome the crisis



Substantive crisis

1. A brief history
2. Current crisis
 - A. Substantive crisis**
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

- **All-Union, 1962, grade 9 of 10:** Given a square table $N \times N$ with odd N , its each cell containing ± 1 , for every horizontal and vertical row we calculate the product of the numbers in this row. Prove that the sum of obtained $2N$ numbers is not equal to 0.

1. A brief history

2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis

4. A special experience

5. Ways to overcome the crisis



Substantive crisis

- **All-Union, 1962, grade 9 of 10:** Given a square table $N \times N$ with odd N , its each cell containing ± 1 , for every horizontal and vertical row we calculate the product of the numbers in this row. Prove that the sum of obtained $2N$ numbers is not equal to 0.

17. Пусть p_1, p_2, \dots, p_n — произведения по строкам, q_1, q_2, \dots, q_n — по столбцам. Тогда $p_1 p_2 \dots p_n = q_1 q_2 \dots q_n$ — мы двумя способами вычисляем произведение всех чисел в таблице. Значит, четность количества -1 среди p_1, p_2, \dots, p_n та же, что и среди q_1, q_2, \dots, q_n , т. е. всего среди $2n$ чисел $p_1, p_2, \dots, p_n, q_1, q_2, \dots, q_n$ четное число -1 и тем самым четное число $+1$. Но тогда число тех и других различно (так как n нечетно) и потому сумма $p_1 + p_2 + \dots + p_n + q_1 + q_2 + \dots + q_n$ не равна 0.

∇ Эта сумма может отличаться от $2n$ лишь на число d , кратное 4. Интересно, построив соответствующие примеры, выяснить, для любого ли $d = 4k$, $|k| < n/2$, сумма может равняться $2n - d$.

1. A brief history
2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

1. A brief history
2. Current crisis
 - A. Substantive crisis**
 - B. Format crisis
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

- **All-Russian, 2015, grade 9 of 11:** Given $N > 8$ different non-negative numbers, each less than 1, with the property that for every 8 numbers there exists 9th such that the sum of these nine numbers is integer, determine how many numbers could there be at least?

1. A brief history
2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Substantive crisis

- **All-Russian, 2015, grade 9 of 11:** Given $N > 8$ different non-negative numbers, each less than 1, with the property that for every 8 numbers there exists 9th such that the sum of these nine numbers is integer, determine how many numbers could there be at least?

1. A brief history
2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Ясно, что при $N = 9$ требуемое возможно – достаточно написать на доске 9 различных положительных чисел с единичной суммой. Покажем, что при $N > 9$ требуемое невозможно.

Предположим противное; обозначим через S сумму всех чисел на доске. Выберем на доске произвольные числа $\alpha_1, \alpha_2, \dots, \alpha_7$ с суммой T ; пусть A – множество всех остальных чисел на доске. По условию, для любого числа $\beta \in A$ найдётся такое отличное от него число $\gamma \in A$, что число $T + \beta + \gamma$ целое. Скажем, что число γ *соответствует* числу β . Заметим, что такое число γ единственно. Действительно, если бы нашлось другое число $\gamma' \in A$, для которого сумма $T + \beta + \gamma'$ целая, то число $\gamma - \gamma' = (T + \beta + \gamma) - (T + \beta + \gamma')$ также было бы целым. Но это невозможно, ибо $0 < |\gamma - \gamma'| < 1$.

В частности, отсюда следует, что β соответствует числу γ . Значит, все числа в A разбииваются на пары чисел $(\beta_1, \gamma_1), \dots, (\beta_l, \gamma_l)$ соответствующих друг другу. При этом $l > 1$, так как $N = 7 + 2l > 9$.

Рассмотрим сумму $\Sigma = (T + \beta_1 + \gamma_1) + (T + \beta_2 + \gamma_2) + \dots + (T + \beta_l + \gamma_l)$. Σ – целое число. С другой стороны, каждое число из A входит в Σ ровно по разу; значит,

$$\Sigma = lT + (S - T) = S + (l - 1)T, \text{ откуда } l = \frac{\Sigma - S}{l - 1}.$$

Выбрав теперь на доске числа $\alpha_2, \alpha_3, \dots, \alpha_8$ и обозначив их сумму через T' , аналогично получаем, что $l' = \frac{\Sigma' - S}{l' - 1}$ при целом l' .

$$\Sigma. \text{ Значит, } \alpha_1 - \alpha_8 = \frac{\Sigma - S}{l - 1} - \frac{\Sigma' - S}{l' - 1} = \frac{\Sigma - \Sigma'}{l - 1}.$$

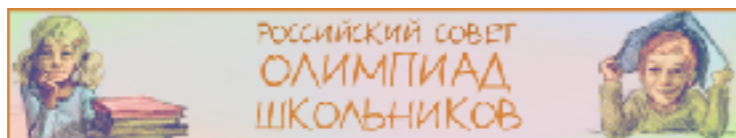
Так как α_1 и α_8 могли быть любыми двумя числами на доске, получаем, что разность каких-либо двух чисел на доске имеет вид $\frac{k}{l - 1}$ при целом k .

Пусть m – наименьшее число на доске. Тогда на доске могут присутствовать лишь числа $m, m - \frac{1}{l - 1}, \dots, m - \frac{l - 2}{l - 1}$ (все

большие числа будут уже не меньше 1) – всего l чисел. Однако общее количество чисел на доске равно $N = 7 + 2l > k$; значит, они не могут быть различными. Противоречие.

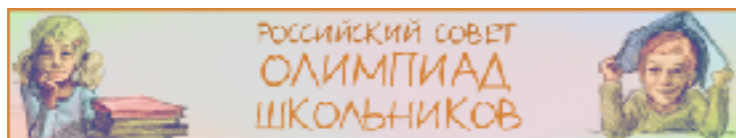
Format crisis

1. A brief history
2. Current crisis
 - A. Substantive crisis
 - B. Format crisis**
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Format crisis

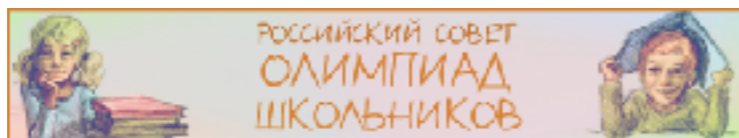
- Contests instead of entrance examinations
 1. A brief history
 2. Current crisis
 - A. Substantive crisis
 - B. Format crisis**
 - C. Ethical crisis
 3. Measures to meet the crisis
 4. A special experience
 5. Ways to overcome the crisis



Format crisis

- Contests instead of entrance examinations
- Special courses for solving Olympiad problems

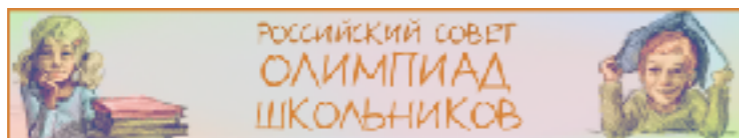
1. A brief history
2. Current crisis
 - A. Substantive crisis
 - B. Format crisis**
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Format crisis

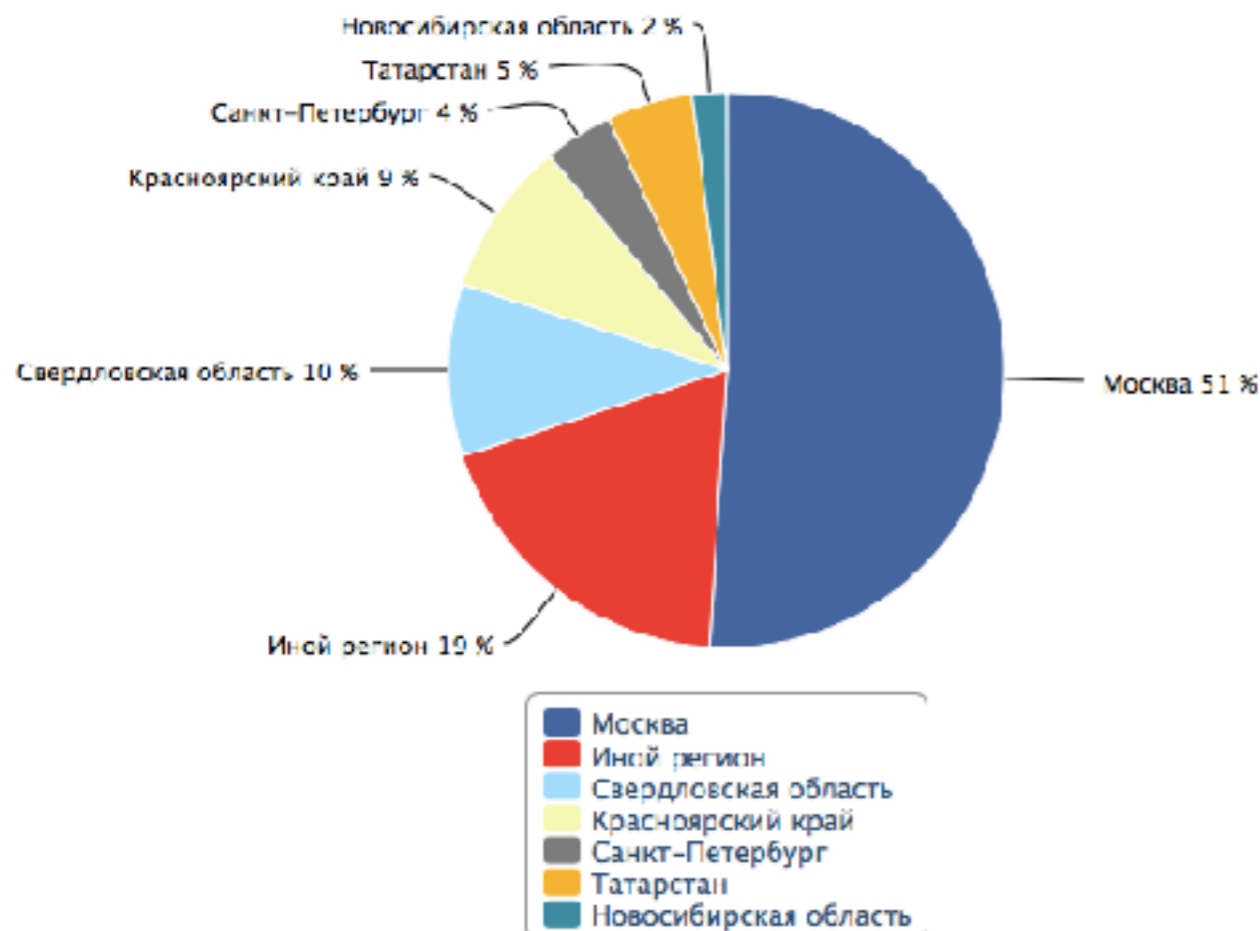
- Contests instead of entrance examinations
- Special courses for solving Olympiad problems
- Parents start to train their children too early

1. A brief history
2. Current crisis
 - A. Substantive crisis
 - B. Format crisis**
 - C. Ethical crisis
3. Measures to meet the crisis
4. A special experience
5. Ways to overcome the crisis



Ethical crisis

- The profit from the winning goes not only to the participant, but also to his coach and his school, thus impacting the relations



1. A brief history

2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis

4. A special experience


5. Ways to overcome the crisis



Measures to meet the crisis

A. Inviting researchers,
organising workshops



Летняя школа
«СОВРЕМЕННАЯ МАТЕМАТИКА» 

1. A brief history

2. Current crisis

A. Substantive crisis 

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis

4. A special experience

5. Ways to overcome the crisis



Measures to meet the crisis

B. Arranging the List and assigning the levels

#	Название олимпиады по математике	№ в перечне	Профиль	Уровни
1	Всероссийская олимпиада школьников «Нанотехнологии - прорыв в будущее»	7	нанотехнологии	1
2	Макрорегиональная олимпиада школьников «Высшая проба»	28	математика	1
3	Макрорегиональная олимпиада школьников по математике и криптографии	33	математика	1
4	Московская олимпиада школьников	41	математика	1
5	Олимпиада школьников «Ломоносов»	53	математика	1
6	Олимпиада школьников «Покори Воробьёвы горы!»	55	математика	1
7	Олимпиада школьников Санкт-Петербургского государственного университета	60	математика	1
8	Санкт-Петербургская олимпиада школьников по математике	77	математика	1
9	Турнир городов	84	математика	1
10	Всесибирская открытая олимпиада школьников	13	математика	2
11	Московская олимпиада школьников	29	математика	2
12	Макрорегиональная олимпиада школьников на базе ведомственных образовательных учреждений	31	математика	2
13	Объединённая межвузовская математическая олимпиада школьников	43	математика	2
14	Объединённая международная математическая олимпиада «Формула Единства» / «Третье тысячелетие»	44	математика	2
15	Олимпиада Курчатова	46	математика	2
16	Открытая олимпиада школьников по математике	67	математика	2
17	Страславская физико-математическая олимпиада школьников «Росатом»	71	математика	2
18	Турнир имени М. В. Ломоносова	85	математика	2
19	Турнир имени М. В. Ломоносова	85	лингвистика	2
20	Олимпиада школьников «Физтех»	87	математика	2
21	Всероссийский конкурс научных работ школьников «Юниор»	10	математика	3
22	Макрорегиональная олимпиада школьников «Будущие исследователи – будущее науки»	27	математика	3
23	Олимпиада школьников «САММАТ»	29	математика	3
24	Олимпиада по дискретной математике и теоретической информатике	49	математика	3
25	Олимпиада школьников «Надежда энергетике»	84	математика	3
26	Олимпиада южарской математической школы	61	математика	3
27	Северо-Восточная олимпиада школьников	63	математика	3
28	Открытая Олимпиада Университета Иннополис для школьников	65	математика	3
29	Макрорегиональная отраслевая олимпиада «Паруса надежды»	72	математика	3

1. A brief history

2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis

3. Measures to meet the crisis

4. A special experience

5. Ways to overcome the crisis



Measures to meet the crisis

C. Dividing the scores, having the right to choose the school

1. A brief history

2. Current crisis

A. Substantive crisis

B. Format crisis

C. Ethical crisis



3. Measures to meet the crisis

4. A special experience

5. Ways to overcome the crisis



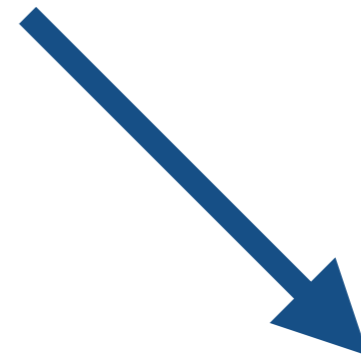
A special experience

Mathematical all-around



Individual
and team
competitions

Algebra & NT
Geometry
Combinatorics & logics
Mathematical race
Team Olympiad



Popular
lectures

Every member
of the
methodological
board delivers
a lecture on his
favourite topic

Everyone
is a winner

Most of the
participants
gain awards
in one of the
competitions

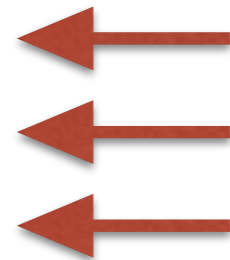
Experimental
round

New forms of
competitions
Arbitrary teams

Ways to overcome the crisis

- A. Cooperation between researchers and Olympiad managers
- B. Understanding the true sense of Olympiads
- C. Reducing the role of the school and the teacher

- 1. A brief history
- 2. Current crisis
 - A. Substantive crisis
 - B. Format crisis
 - C. Ethical crisis
- 3. Measures to meet the crisis
- 4. A special experience
- 5. Ways to overcome the crisis**



Thank you for your attention!

Vladimir Sharich / Шарич Владимир Златкович

sharich@mathschool.ru

«Foxford» online school <http://foxford.ru>

«Mathematical school» society <http://mathschool.ru>

«Phystech-lyceum» school <http://ftl.name>



QUESTIONS



QUESTIONS

A. How to bring new fresh ideas to competition problems and make them more like science?



QUESTIONS

- A. How to bring new fresh ideas to competition problems and make them more like science?
- B. What is the real meaning and purpose of mathematical competitions?



QUESTIONS

- A. How to bring new fresh ideas to competition problems and make them more like science?
- B. What is the real meaning and purpose of mathematical competitions?
- C. What kind of recognition should be implemented for teachers and schools?

